647510 **BSc3MatC301x** **Seat No :________**

B.Sc. Semester - 3 (*CBCS*) Examination
Oct/Nov - 2024 [As Per Syllabus Year June-2019]
Mathematics-M301(Core)

Time: 2:30 Hours **Marks: 70**

Instructions:
1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1(A) Answer any two out of three. (10)

(1) If S is a non empty subset of vector space V then prove that Sp(S) is a subspace of V.

(2) Prove that the set of vectors $\{\bar{v}_1, \bar{v}_2, \dots, \bar{v}_n\}$ is L.D. iff there exist a vector $\bar{v}_k$, $2 \le k \le n$, which is a linear combination of its preceding vectors $\bar{v}_1, \bar{v}_2, \dots, \bar{v}_{k-1}$.

(3) Set $A = \{ (1,-2,5),\ (2,1,-1),\ (3,-1,b) \}$ is a subset of vector space $\mathbb{R}^3$. If set A is linearly dependent then find the value of b.

Que.1(B) Answer any one out of two. (04)

(1) Check whether vector $(1, 0, -1) \in$ Sp(A) where set A={(2,1,0), (-1,0,1), (0,1,2)}.

(2) Check whether set $A = \left\{\sin x\ ,\ \cos x\ ,\cos\left(x - \frac{\pi}{4}\right)\right\}$ of real vector space C[0, 2π] is linearly dependent or linearly independent.

Que.2(A) Answer any two out of three. (10)

(1) State and prove existence theorem for basis.

(2) Define finite dimensional vector space V and prove that any two basis of V have the same number of vectors.

(3) Consider two subspaces
$W_1 = \{ (x-y+z,\ x+y-z,\ y+z,\ 2x+4y+4z) \ /\ x,y,z \in \mathbb{R} \}$ and
$W_2 = \{ (2y-4z+w,\ y+z,\ y-z,\ w) \ /\ y,z,w \in \mathbb{R} \}$ of $\mathbb{R}^4$.
Find dim(W_1+W_2), dimW_1, dimW_2, dim($W_1 \cap W_2$) .

Que.2(B) Answer any one out of two. (04)

(1) Extend subset $A = \{(1,1,1), (2,0,0)\}$ of vector space $\mathbb{R}^3$ to form a base of $\mathbb{R}^3$.

(2) Prove that set $\{1-x, 1+x, 1-x^2\}$ is a base of $P_2(\mathbb{R})$. Find coordinates of $3 + 7x + 2x^2$ with respect to this base.

Que.3(A) Answer any two out of three. (10)

(1) (i) In usual notations, prove that $div(\phi\bar{f}) = \phi div\bar{f} + \bar{f}.grad\phi$.

(ii) For a scalar function ϕ on D, Prove that curl(grad ϕ) = $\bar{0}$.

(2) Prove that $\nabla^2(\phi\Psi) = \phi\nabla^2\Psi + 2\nabla\phi\Psi + \Psi\nabla^2\phi$

(3) Prove that $\dfrac{1}{r}$ satisfies the Laplacian equation. Also prove that if r = $|\bar{r}|$ Then Curl(grad r^n)= $\bar{0}$ where $\bar{r}$ = (x, y, z).

Que.3(B) Answer any one out of two. (04)

(1) If $\bar{f}$ = (x^3,y^3,z^3) then prove that curl $\bar{f}$ = $\bar{0}$ and grad(div $\bar{f}$)= 6$\bar{r}$.

(2) In usual notations, prove that div(r^n $\bar{r}$) = (n+3) r^n

Que.4(A) Answer any two out of three. (10)

(1) Find the area of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, (a>b)$ by double integral.

(2) Evaluate: $\iiint\limits_R \frac{dxdy}{\sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}}}$ where R is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

(3) Define triple integral. Find the volume of a sphere having radius a.

Que.4(B) Answer any one out of two. [4] (04)

(1) Change the order of integration in the integral $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xydydx$ and hence evaluate it.

(2) Transform into spherical coordinates:
$\iiint\limits_R (x^2 y)dxdydz$ where R is $x^2+y^2+z^2 \le a^2$.

Que.5(A) Answer any two out of three. (10)

(1) State and prove Green's theorem for a plane.

(2) Verify divergence theorem for $\bar{V} = x\hat{\imath} + y\,\hat{\jmath} + z\,\hat{k}$, and S is a sphere $x^2+y^2+z^2=1$.

(3) Define Beta function. Prove that $\beta(p,q) = \int_0^\infty \frac{y^{p-1}}{(1+y)^{p+q}}\,dy$, where $p,q>0$.

Que.5(B) Answer any one out of two. (04)

(1) Evaluate: $\int_0^1 \frac{x^5}{\sqrt{1-x^4}}dx.$.

(2) Verify Stokes theorem for $\overline{F} = x^2\bar{i} + xy\,\bar{j}$ where S is a square whose sides are $x=0,\ y=0,\ x=a,\ y=a$ where $z=0$.

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